Estimation of the MOND Critical Gravitational Acceleration Scale $\mathbf{a_o}$ by the Method of Dimensions and Consequences for Galaxy Development in the Early Universe.

Geoff Kirby

6 College Lane, Weymouth, Dorset, DT4 7LP, UK

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Summary

Forty years ago, Mordehai Milgrom published a series of groundbreaking papers introducing a new theory of gravity called Modified Newtonian Dynamics (MOND). This largely eliminated the need for postulating 'Dark Matter' to explain the observations that stars in orbit within galaxies travel at velocities much higher than predicted by standard Newtonian Dynamics and largely have velocities independent of distance from the galaxy core. Despite a huge intellectual and financial investment in the 'Dark Matter' postulate, MOND remained a viable alternative theory for four decades.

MOND introduces a parameter $\mathbf{a_o}$ which is a critical level of acceleration in a gravitational field below which Newtonian Dynamics substantially fails.

This paper uses the Method of Dimensions to link $\mathbf{a_o}$ in a gravitational field to the host galaxy's red shift $\mathbf{z}$ as well as to the Planck Length $\mathbf{P_L}$, Planck Time $\mathbf{P_T}$, Planck Mass $\mathbf{P_M}$, Boltzmann's constant, $\mathbf{k}$, and the effective temperature of the Cosmic Microwave Background Temperature (CMBT) measured local to the Earth, $\boldsymbol{\theta}$.

It is postulated here that

$$\mathbf{a_o} = 6\pi^2\ \mathbf{P}_L^{-3}\,\mathbf{P}_T^{2}\,\mathbf{P}_M^{-2}\ (\mathbf{k}\boldsymbol{\theta})^2\ \ (1 + \mathbf{z})^2\ \mathrm{m/s^2} \qquad ...(1)$$

in the gravitational field of a galaxy with redshift $\mathbf{z}$.

This gives a value for $\mathbf{a_o}$ equal to 1.22×10^{-10} m/s^2 for $\mathbf{z}$ close to zero which applies to local galaxies used to determine the value for $\mathbf{a_o}$. This is within the uncertainty of the measured value of about 1.2×10^{-10} m/s^2.

The above equation can be simplified as

$$\mathbf{a_o}(\mathbf{z}) = \mathbf{a_o}(0)(1 + \mathbf{z})^2\ \mathrm{m/s^2} \qquad ...(2)$$

This equation predicts that, for example, in a galaxy having a redshift of ten, the acceleration experienced by stars is significantly larger by an order of magnitude than that observed for our, and other local galaxies. This predicts that the orbital velocity of stars in high redshift galaxies is about three times faster than in local, similar mass, galaxies.

This might explain the recent JWST observations that galaxies developing in the very early universe appear to have condensed, matured and formed supermassive 'Black Holes' substantially faster than predicted by measures of their star radial velocities using conventional cosmological theories.

Introduction

Fritz Zwicky *et al* [1] discovered that galaxies moving within galaxy clusters had a significantly higher velocity than predicted by using Newtonian dynamics. Also, Vera Rubin *et al* [2] discovered that the velocities of stars in galaxies is much faster than can be explained by Newtonian dynamics, see fig. 1. This has led to the hypothesis that the universe is non-uniformly filled with 'Dark Matter' comprised of particles which do not interact with ordinary matter but which can exert a gravitational force. These assumed particles are not consistent with the Standard Model of Particle Physics.

Despite many experimental searches, no convincing evidence has been found for the postulated dark matter particles.

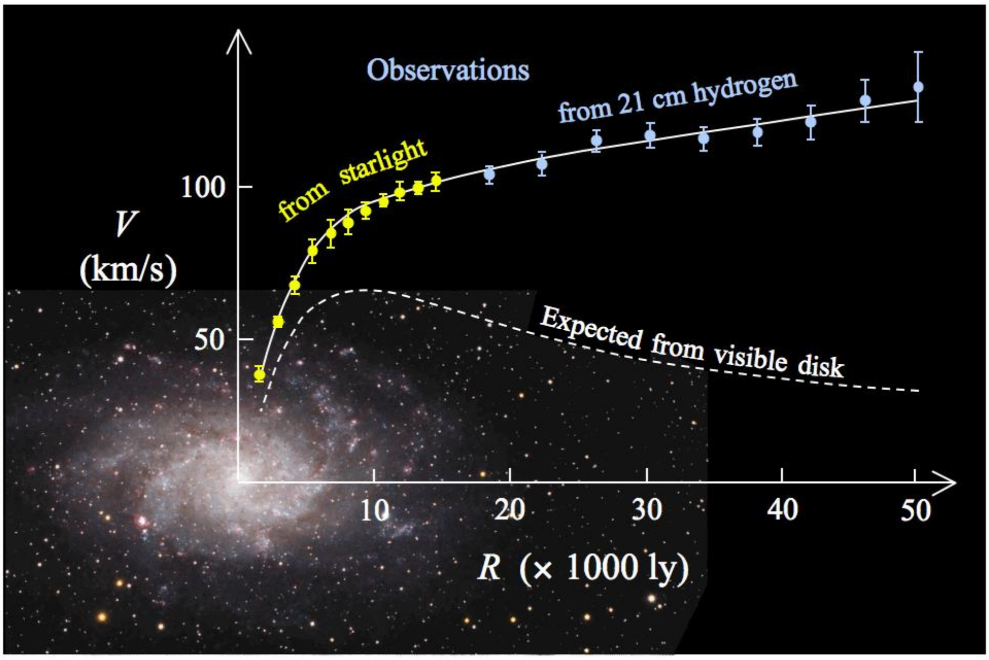

Fig. 1 The discrepancy between predicted and observed star velocities in galaxy M33. [3]

The failure to identify dark matter particles and the inability to incorporate such particles into the highly successful Standard Model of particle physics, encouraged the invention in 1983 of an alternative model for gravity known as Modified Newtonian Dynamics (MOND).[4,5,6]

Banik and Zhao have published an 87-page summary of the MOND vs Dark Matter controversy. [7] In that 2022 paper they write

"Our conclusion is that MOND is favoured by a wealth of data across a huge range of astrophysical scales, ranging from the kpc scales of galactic bars to the Gpc scale of the local supervoid and the Hubble tension, which is alleviated in MOND through enhanced cosmic variance."

At the time of writing, MOND is being touted as being invalidated by measurements of relative velocities of far-separated binary stars where the dynamics should be consistent with MOND. [8] However, a different sample of the same Gaia dataset provided good support for MOND. [9]

In the words of Mark Twain “The reports of my death are greatly exaggerated.”

The debate will not be further discussed here. The jury is out and deliberating.

Modified Newtonian Dynamics (MOND)

MOND theory proposes a modification to the conventional version of Newton’s Second Law of Motion

$$\mathbf{F_N = ma} \quad \text{... (3)}$$

where $\mathbf{F_N}$ is the force of gravitation from Newtonian or General Relativity equations, **m** is the mass being acted upon and **a** is the acceleration of the mass being acted upon. $\mathbf{F_N}$ changes to the form

$$\mathbf{F_N = ma^2/a_o} \quad \text{... (4)}$$

when $\mathbf{a \ll a_o}$ where $\mathbf{a_o}$ is a transition level of acceleration. Its value is about $1.2\text{x}10^{-10}$ m/s^2 determined from measurements of the rotation velocities of stars within their parent galaxies. [10]

If we insert the Newtonian force into equation (4) and set the acceleration of the acted-upon mass as $\mathbf{V^2/R}$, where **V** is the radial velocity of a star in its orbit and **R** is the radius from the galaxy’s centre of gravitating mass, then we obtain

$$\mathbf{V^4 = GMa_o} \quad \text{...(5)}$$

where **G** is Newton’s Gravitational Constant and **M** is the mass of the gravitating object.

Hence, MOND predicts that in the outer reaches of galaxies, the velocities of stars are essentially constant with range from the galaxy’s centre – which is what is generally observed. [11]

Where Does Newtonian Dynamics Break Down?

This table shows the approximate ranges from a central body at which the Newtonian gravitation acceleration reaches $\mathbf{a_o}$ m/s^2.

Gravitating Object	Mass kg	Transition Range (a_o = 1.2x10^{-10} m/s^2)			
		metres	Astronomical Units	Light Years	kParsecs
Earth	5.97x10^{24}	1.82x10^{12}	12.18	0.0001926	5.91x10^{-8}
Jupiter	1.89x10^{27}	3.25x10^{13}	217	0.00343	1.05x10^{-6}
Sun	1.99x10^{30}	1.05x10^{15}	7031	0.111	3.41x10^{-5}
Galactic Black Hole	8.15x10^{36}	2.13x10^{18}	1.42x10^{7}	225	0.069
M87 Black Hole	9.94x10^{39}	7.43x10^{19}	4.97x10^{8}	7861	2.41

Table 1. Transition Ranges from Newtonian to MOND Gravitation regions.

It can be seen that the transition range from Earth (and from Venus which has a similar mass) is about 12 Astronomical Units (AU) which is less than the distance of Uranus (19 AU) and all the solar system bodies beyond. Thus, Uranus, Neptune and the trans-Neptunian dwarf planets will be perturbed in their orbits by the gravitational attractions of Earth and Venus more than predicted using Newtonian dynamics if MOND is correct. However, the effects are utterly negligible compared with the domination by the outer gas giant planets so this is an unlikely source of confirmatory evidence for MOND.

The transition to MONDian dynamics for the Sun is about 3% of the way to the nearest star, Proxima Centauri so that all the solar system's bodies are in the regime of the Sun's Newtonian dynamics.

However, when it comes to the massive black hole at the centre of our galaxy and the super-massive black hole in M87, the majority of stars are well in the range at which MOND dynamics apply which is why galaxies generally have stars rotating at approximately constant speed irrespective of distance from their galaxy's core.

MOND is successful in quantitatively explaining different features of galaxy dynamics. However, this is not the place to argue the merits of MOND compared with rival theories such as Dark Matter. There is a large literature devoted to that debate. [12]

What Is a_0?

Rosco has stated: [13]

"The irrefutable successes of MOND are predicated upon the idea that a critical gravitational acceleration scale, $\boldsymbol{a}_0$, exists. But, beyond its role in MOND, the question: *'Why should a critical gravitational acceleration scale exist at all?'* remains unanswered."

In 1983 Milgrom suggested that the constant, $\boldsymbol{a}_0$, is proportional to the product of the speed of light, **c**, and Hubble's Constant, $\mathbf{H_0}$. [14, 15, 16, 17, 18]

It has been noted that [19]

$$\mathbf{a_0} \approx \mathbf{cH_0}/2\pi \qquad \text{...(6)}$$

and It has also been noted that

$$\mathbf{a_0^2} \approx \mathbf{\Lambda} \qquad \textbf{...(7)}$$

where $\mathbf{\Lambda}$ is the cosmological constant. [20]

Equation 44 of a paper by Schlatter and Kastner [21] specifically links $\mathbf{cH_0}$ and $\mathbf{\Lambda}$ within the assumption implied in the theory of Entropic Gravity. These predictions may be insightful into the interpretation of $\mathbf{a}_0$ in terms of the universe's fundamental constants.

However, it is by no means clear to the author why Newtonian dynamics should break down at a particular point in space-time in a manner dependent on the rate of expansion of the universe. Objects that are gravitationally bound do not take part in the expansion of the universe. The orbits of planets around our Sun have not grown in size as distant galaxies are carried away from us at an accelerating rate by the so-called 'Dark Energy'. [22]

Imagine a location in space-time where the gravitational acceleration is equal to Milgrom's parameter $\mathbf{a_o}$ = 1.2x10^{-10} m/s^2. Surely the value of $\boldsymbol{a}_0$ will depend on parameters local to spacetime where the gravitational acceleration is equal to $\mathbf{a}_0$? These local parameters are here drawn from Quantum Theory and the all-pervading field of photons from the Cosmic Microwave Background Radiation (CMBR) which is a remnant of the 'Big Bang'.

Dimensional analysis has been used to suggest how $\mathbf{a_o}$ might depend on Quantum Theory and the afterglow of the 'Big Bang'.

The most obvious application of dimensional analysis is to investigate whether $\boldsymbol{a}_0$, which has dimensions of acceleration, can be plausibly related to the Planck Length $P_L = \sqrt{\left(\frac{\hbar G}{c^3}\right)}$ and Planck Time $P_T = \sqrt{\left(\frac{\hbar G}{c^5}\right)}$ by the ratio $\frac{P_L}{P_T^2}$**.** This ratio is equal to 5.6x10^{51} m/s^2 which is about 61 orders of magnitude larger than $\boldsymbol{a}_0$! This is not, by any means, the worst discordance between a prediction and the reality – a theory for 'Dark Energy' predicts a value about 10^{120} too large! [23]

We need to involve more variables and those proposed here are the Planck Mass, $\mathbf{P_M}$ which is equal to $\sqrt{\left(\frac{\hbar c}{G}\right)}$ and the energy of photons involved in the CMBR, $\mathbf{k\theta}$, where $\mathbf{k}$ is Boltzmann's constant and $\boldsymbol{\theta}$ is the mean temperature of the CMBR observed from Earth, namely 2.725 Kelvin. The value of these parameters predicted to be involved in the value of $\mathbf{a_o}$ are as shown below.

$\mathbf{P_L}$ is the Planck length $\sqrt{\left(\frac{\hbar G}{c^3}\right)}$ = 1.616x10^{-35} m ...(8a)

$\mathbf{P_T}$ is the Planck time $\sqrt{\left(\frac{\hbar G}{c^5}\right)}$ = 5.391x10^{-44} s ...(8b)

$\mathbf{P_M}$ is the Planck mass $\sqrt{\left(\frac{\hbar c}{G}\right)}$ = 2.176x10^{-8} kg ...(8c)

k is Boltzmann's constant =1.3806x10^{-23} J/K ...(8d)

$\boldsymbol{\theta}$ is the local CMBT of the universe, 2.725 K ...(8e)

The proposition of this paper is that Milgrom's parameter can be represented by the equation

$$\mathbf{a_o} = N\, \mathbf{P}_L^A\, \mathbf{P}_T^B\, \mathbf{P}_M^C\, (\mathbf{k\theta})^D \quad \mathrm{m/s^2} \qquad ...(9)$$

where N is an unknown numerical factor and A, B, C and D are numerical values determined by the requirement that $\mathbf{a_o}$ has the dimensions of acceleration.

There are insufficient parameters to determine unique values for the indices A, B, C and D. So, we relate A, B and C to D through the following relationships.

$$A = 1 - 2D, \quad B = 2D - 2, \quad C = -D \qquad \text{...(10)}$$

The value of $\mathbf{P}_L^A \mathbf{P}_T^B \mathbf{P}_M^C\ (\mathbf{k\theta})^D$, which we designate **Z**, has been plotted against the values of 'D' in the chart below.

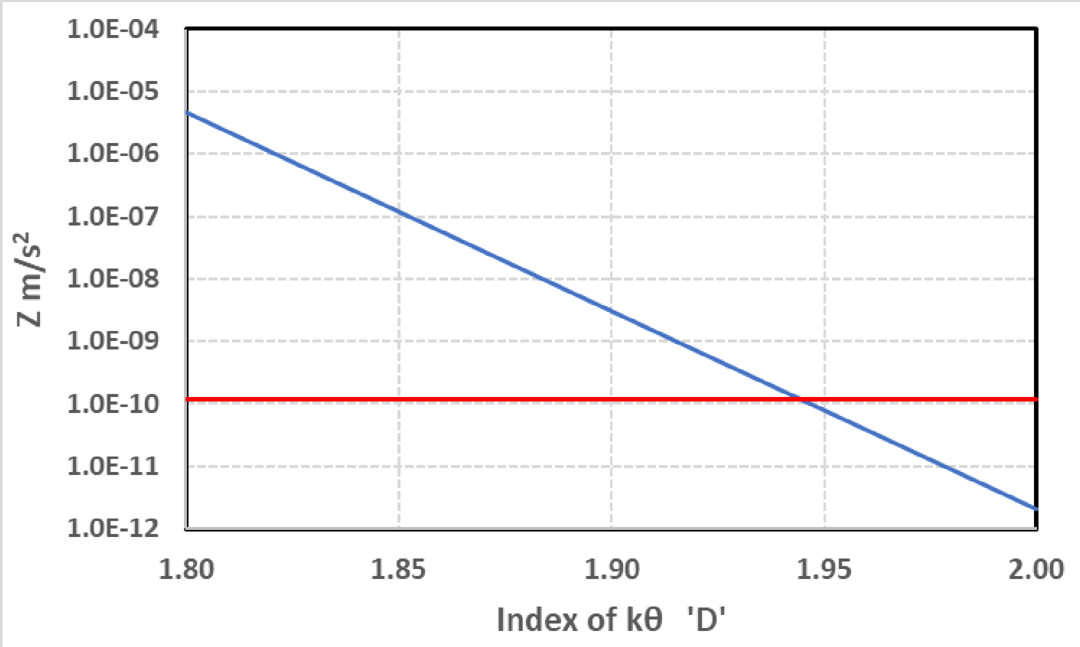

*Fig. 2 Dependence of '**Z**' on 'D'*

The value for Milgrom's parameter $\mathbf{a_o}$ is shown by the horizontal line on the above chart.

It is seen that the parameter '**Z**' is equal to Milgrom's parameter when D is approximately 1.945 for which condition N = 1 in equation 9.

The observation that the index 'D' is very close to the whole number 2 is suggestive that perhaps 'D' *is* 2. Setting D equal to 2 gives a value of N equal to 58 in equation 9.

Is it a coincidence that $6\pi^2 = 59.2$ which is well within the experimental uncertainty band for determining the Milgrom parameter $\mathbf{a}_0$?

It is proposed that D = 2 and N = $6\pi^2$ are acceptable assumptions giving

$$\mathbf{a}_0 = 6\pi^2\ \mathbf{P}_L^{-3}\ \mathbf{P}_T^{2}\ \mathbf{P}_M^{-2}\ (\mathbf{k\theta})^2 = 1.22\text{x}10^{-10}\ \text{m/s}^2 \qquad \text{...(11)}$$

This is a conclusion from this paper.

Consequences For Early Galaxy Development

The foregoing analysis suggests that the Milgrom Parameter $\mathbf{a_o}$ as measured for local galaxies, has a value proportional to the square of the temperature of the local CMBT, $\boldsymbol{\theta}$, which is 2.725 K. Noterdaeme, P., *et al* have measured the value of the CMBT for galaxies with high redshifts based on observations carried out at the European Southern

Observatory (ESO). [24] These analyses confirm the expectation from the adiabatic model of universal expansion that

$$\boldsymbol{\theta} = 2.725\ (1 + \mathbf{z})\ \mathrm{K} \qquad \text{...(12)}$$

where **z** is the redshift of a distant gravitating object.

This leads to a simple relationship between Milgrom's Parameter and the redshift of a distant gravitating mass, which is

$$\mathbf{a_o}(\mathbf{z})/\mathbf{a_o}(0) = (1 + \mathbf{z})^2 \qquad \text{...(13)}$$

The means that for high redshifted, young objects at great distances, the acceleration within these galaxies may be higher than for the same object being local with a redshift of close to zero.

Equation 13 shows that the asymptotic orbital velocities of stars in the MOND region around gravitating bodies is given by

$$\mathbf{V}^4 = \mathbf{GMa_o}\ (1 + \mathbf{z})^2\ \mathrm{m/s}^2 \qquad \text{...(14)}$$

Thus, the asymptotic orbital velocities of stars is faster in high red shift galaxies. Not allowing for this effect would cause the mass of high red shift galaxies to be overestimated and this may account for the puzzling observations by the JWST that galaxies in the very early universe have higher masses and more massive central 'Black Holes' than predicted by the current model for galaxy development in the early universe.

Returning to the fundamental relations of MOND between acceleration force and local acceleration, i.e., equations 3 and 4, we now have

$$\mathbf{F_N} = \mathbf{ma} \qquad \text{... (15)}$$

when $\mathbf{a} >> \mathbf{a_o}$ and

$$\mathbf{F_N} = \mathbf{ma}^2/\mathbf{a_o}(1 + \mathbf{z})^2 \qquad \text{... (16)}$$

when $\mathbf{a} << \mathbf{a_o}$

Fig. 3 shows the relationship between $\mathbf{F_N/m}$ and **a** for redshifts from local (z = 0) to high (z ≈ 10).

The formula covering the transition from Newtonian to MONDian dynamics is [25]

$$\mathbf{F_N} = \mathbf{ma}\sqrt{\frac{1}{1+(\mathbf{a_0}/\mathbf{a})^2}}$$

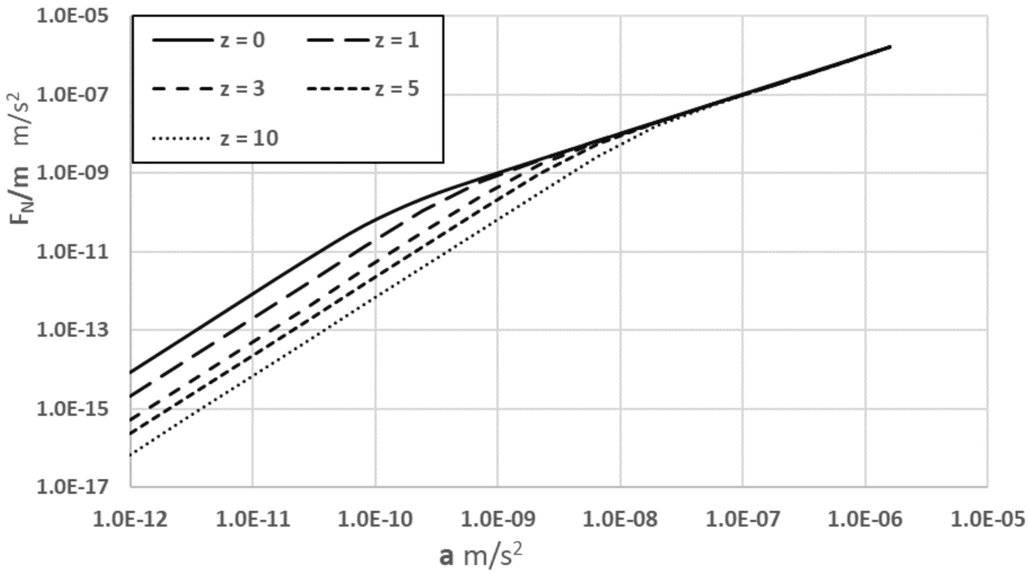

Fig. 3 Dependence of the gravitating force $\mathbf{F_N}$ *and local acceleration* $\mathbf{a}$ *as a function of red shift*

It can be seen that, deep in the MONDian regime, the force $\mathbf{F_N}$ can be two orders of magnitude weaker for a given gravitational acceleration for galaxies with redshift around ten when compared with local galaxies.

Supporting Evidence

Bekenstein and Sagi have suggested that $\mathbf{a_o}$ could vary with redshift. [26]

Measurements have been achieved of stellar orbital speeds of stars in a galaxy with $\mathbf{z}$ = 9.1 corresponding to a time about 550 million years after the 'Big Bang'. The high values for star rotation have been assumed to be associated with larger supermassive black holes and more massive galaxies than explained by current standard models of early galaxy development. Perhaps these observations are better explained by MOND with a value for $\mathbf{a_o}$ which was higher at high redshifts?

For example, galaxy GN-z11 observed by the JWST using gravitational lensing appears to have a supermassive 'Black Hole', based on star velocity measurements, at its heart despite the galaxy being observed when the universe was only about 400 million years old. [27]

With the JWST now fully operational evidence should now become abundant for testing the proposition here that

$$\mathbf{a_o}(\mathbf{z})/\mathbf{a_o}(0) = (1 + \mathbf{z})^2 \qquad ...(17)$$

and that the asymptotic orbital velocities of stars ($\mathbf{V}$) around gravitating masses is higher than expected from conventional MOND theory when applied at large red shifts such that

$$\mathbf{V} = \sqrt{(1+z)} \qquad ...(18)$$

Thus, evidence may become available to test MOND on galaxies with redshifts high enough to detect the anomalous star orbital velocities predicted by equation 14.

Conclusions

Insight into the physical dependence of $\mathbf{a_o}$ in the MOND interpretation of gravitational acceleration on the properties of localised space-time has been gained using dimensional analysis. It has been assumed that $\mathbf{a_o}$ is dependent on quantum parameters and the Cosmic Microwave Background Temperature. This gives rise to the expression

$$\mathbf{a}_0 = 6\pi^2\ \mathbf{P_L^{-3}}\ \mathbf{P_T^{2}}\ \mathbf{P_M^{-2}}\ (\mathbf{k\theta})^2 = 1.22\mathrm{x}10^{-10}\ \mathrm{m/s^2} \qquad \text{...(19)}$$

which can also be written

$$\mathbf{a_o(z)}\ /\ \mathbf{a_o}(0) = (1 + \mathbf{z})^2\ \mathrm{m/s^2} \qquad \text{...(20)}$$

where **z** is the redshift of the galaxy under consideration.

This shows that MOND predicts, under the assumptions in this paper, that gravitational acceleration was greater in high redshift galaxies in the early universe. Typically, gravitational attraction would have been an order of magnitude greater in young galaxies with redshift of around ten. This would have resulted in early epoch galaxies forming and maturing more rapidly than expected from the Standard Cosmological Model.[28]

Biographical Summary

Geoff Kirby was born in the 1930s and graduated from London University with a First-Class Honours BSc in Physics. He is one of a small and rapidly diminishing group who have attended a lecture given by Neils Bohr. On retirement in 1992 he was the Head of the Royal Navy's Oceanographic Research Programme. He then spent five years studying for an Open University BSc in Environmental Studies with an extra year on the History of Mathematics all just for fun. This paper is an example of his determination to keep his ancient brain active and gainfully employed as he proceeds through his life's ninth decade.

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[10] I have named this "Milgrom's Parameter". The current estimate is 1.2×10^{-10} m/s^2

[11] This assumes that the gravitating mass is located at the core of the galaxy whereas in practice the stars and gas are nonuniformly distributed throughout the disc and halo of the galaxy. This explains why the rotational velocity curve in fig. 1 is not quite constant. However, many galaxies shows stars in orbit with velocities remarkably independent of range from the galactic cores.

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[28] https://www.youtube.com/watch?v=hmkyF1tNFc4